Parameterized and Approximation Algorithms for LOAD COLORING

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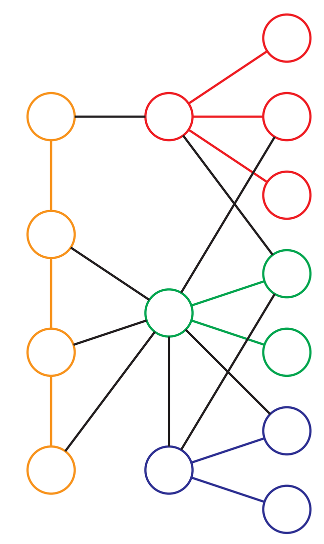
THE c-LOAD COLORING PROBLEM $(c,k)K_2$

c-LOAD COLORING

Input : a graph $G=(V,E)$ and a positive integer k as parameter
Output : $\exists ?\ \varphi : V \to [c], \forall i \in [c], |E(\varphi^{-1}(i))| \geq k$

Given a graph G and an integer k, the c-LOAD COLORING problem asks whether there exists a coloring $\varphi : V \to [c]$ such that for every $i \in [c]$, there are at least k edges with both endvertices colored i.



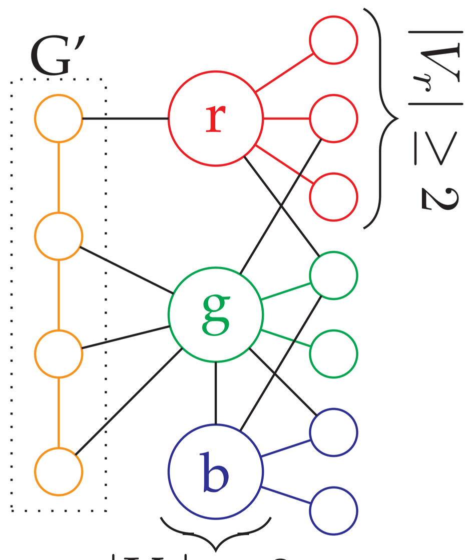
$G \in (4,2)K_2$

RESEARCH RESULTS

Overload. A pair (V_1, V_2) of disjoint vertex sets is an *overload from* $O_{|V_1|,k}$ if

- $\forall v \in V_2$, $N(v) \subseteq V_1$,
- $\forall u \in V_1$, $\exists V_u \subseteq N_{V_2}(u)$ such that $|V_u| \geq k$ and $\forall v \in V_1/u$, $V_u \cap V_v = \emptyset$.

Reduction rule $\mathbf{R_{i,k}}$. If an instance G of c-LOAD COLORING contains an overload $(V_1,V_2) \in O_{i,j}$, $j \geq k$, delete the vertices of $V_1 \cup V_2$ from G and decrease c by i.



The previous colored graph G contains an overload from $O_{3,2}$.
The remaining graph after using the reduction rule $R_{3,2}$ is G'.

This overload structure, crossing the well known crown decomposition and the star-graph, is introduced in our paper [2]. We obtain that:

- Reduction Rules $R_{i,k}$ are safe and polynomial. They may run in $O((cn)^2)$.
- There exists a kernel with less than $2ck$ vertices and less than $6.25c^2k$ edges.
- The problem is minor-bidimensional + The path $P_{c(k+1)}$ is a positive instance.

From these results, we prove the c-LOAD COLORING admits :

$\Rightarrow$ a parameterized algorithm using pathwidth with running time $O^*(c^{ck})$.
$\Rightarrow$ a parameterized algorithm using treewidth with running time $O^*(c^{O(g\sqrt{ck})})$ (where g is the genus of the input).
$\Rightarrow$ an approximation algorithm with constant ratio (namely, $12.5c$).

DOMAIN

Graph Theory. The c-LOAD COLORING problem belongs to *Graph Theory*, a domain crossing discrete mathematics and computer science. The main structure studied in this theory is the *graph*. Inter alia, a *graph* may be used in social, physical, biological or information systems to model pairwise relations between objects. It may represent data organization, networks of communication, computational devices or chemical compounds. The development of efficient algorithms to handle *graphs* is therefore of major interest.

Parameterized Complexity. The efficiency of an algorithm is studied and measured by the *Computational Complexity Theory*. The *Parameterized Complexity* is a branch of this theory that focuses on classifying computational problems according to their inherent difficulty with respect to multiple parameters of the input of the algorithm. For more elaborate introductions to parameterized algorithms and complexity, see [3].

TERMINOLOGY

Graph. A *Graph* is an ordered pair $G=(V,E)$, comprising a set V of *vertices* together with a set $E \subseteq V^2$ of *edges* connecting the *vertices*.

Notation. For a set X, $|X|$ is the cardinality of X. For any positive integer p, $[p] = \{1, 2, \ldots, p\}$.

Kernelization is a technique for designing efficient algorithms that achieve their efficiency by a preprocessing stage in which inputs are replaced by a smaller input, called a *kernel*.

INTERESTS

Context. The 2-LOAD COLORING problem was introduced in [1] with applications in broadcast WDM communication networks. It is natural to study the generalization c-LOAD COLORING, which is a subcase of the STAR PACKING problem and, more generally, of the H-PACKING problem $(c,k)H$. It is therefore strongly related to deletion and cover problems.

Complexity. The c-LOAD COLORING problem is NP-Complete and Fixed-Parameter Tractable. While there are many parameterized graph problems which admit kernels linear in the number of vertices, usually only problems on classes of sparse graphs admit kernels linear in the number of edges. We show c-LOAD COLORING is somewhat unusual since it has a linear-edge kernel for every fixed c. This kind of kernel bounds deserves to be more studied since they may imply efficient approximation algorithms.

POETRY

The genius is what makes the stars bright in daylight. (adapted from A. ESQUIROS).

REFERENCES

[1] N. Ahuja, A. Baltz, B. Doerr, A. Privtivy and A. Srivastav, On the Minimum Load Coloring Problem. J. Discr. Alg. 5(3):533-545 (2007).

[2] F. Barbero, G. Gutin, M. Jones, and B. Sheng, Parameterized and Approximation Algorithms for the Load Coloring Problem. In *10th International Symposium on Parameterized and Exact Computation 2015*, LIPIcs, to appear.

[3] R.G. Downey and M.R. Fellows, Foundations of Parameterized Complexity, Springer, 2013.

FUTURE RESEARCH AND OPEN QUESTIONS

Our work opens optimization and complexity questions:

- Does there exist a sub-exponential algorithm for c-LOAD COLORING, whatever the input?
- How to improve the linear-edge kernel bound to match with the lower bound?
- Is our tight linear-vertex kernel bound optimal or does there exist another reduction rules?
- May we use some of our ideas for the general $(c,k)H$ packing problem or for other subcases?