Quantum Walks

(Dated: November 17, 2017)

Supervisor : Giuseppe Di Molfetta (MCF).

Contact details : giuseppe.dimolfetta@lif.univ-mrs.fr, 0646497714.

Place : Laboratoire d'Informatique Fondamentale (LIF), Natural Computing team (CaNa). Scientific environment: The CaNa research group (Pablo Arrighi, Giuseppe Di Molfetta, Kevin Perrot, Sylvain Sen) seeks to capture at the formal level some of the fundamental paradigms of theoretical physics and biology, via the models and approaches of theoretical computer science and discrete mathematics. The group is located in Luminy, Marseille, France, and benefits from a rich scientific environment with the Cellular Automata experts of I2M (Pierre Guillon, Guillaume Theyssier) and the physicists from CPT (Alberto Verga, Thomas Krajewski).

Theme : A *classical Markov Chain* (MC) is a stochastic process that assumes values in a discrete set and obeys the following property: the next state of the chain only depends on the current state, i.e., it is not influenced by the past states. As computer scientists we can see the MC as a direct graph where the states are represented by the vertices and direct edges indicate what the possible next states are. If the probability distribution is know at the time t, we obtain the distribution at time $t+1$ by employing the formula:

$$\overline{p}(t) = M\overline{p}_i(t-1)$$

where M is called the stochastic matrix and the entry M_{ij} is the probability of the walker $p(t)$, who is in vertex x_j, to go to the vertex x_i. From computer science to Biology, we know MC are very useful to describe many phenomena: disease spread, market dynamics, DNA sequences, etc...

The construction of the quantum counterpart of this process, namely *Quantum Walks* [4], can be performed by a process called $quantization$: the state of the system is no longer described by the real nonnegative number p_i, but by a complex vector Ψ, and the stochastic matrix is replaced by a unitary matrix U

$$\Psi(t) = U\Psi(t-1)$$

The main difference that we want to stretch here is that, unitary matrix means that the process is *reversible* ($i.e.$ $\Psi(t-1) = U^{-1}\Psi(t)$ is also true) and then *there is no room for randomness*.

Why are we interested in considering such a generalization? It turns out that QWs are an elegant way to model quantum behavior in nature in several systems (e.g. energy transfer in photosynthesis [3] or quantum matter [1, 2]). Notice that several of these systems live on non trivial topology and discrete geometry as a Simplicial Complex (SC).

The goal : The aim of this internship is to study QWs over some regular SC in two dimensions, e.g. triangular one. The steps to follow towards this aim seem relatively clear to us, yet answering this question, performing numerical simulations, would be the student's main task, and achieving it would already mean a successful internship. Still, once this is done, one may wonder about QWs over simplicial complexes that are less regular and taking into account defects. Moreover the student, during the internship, will profit of an existing and consolidated scientific collaboration between our group and the University of Valencia. Mobility will be encouraged. Notice that, this internship could eventually be extended with a 3-years long PhD, within the same consortium. The internship will funded as legally required.

Prerequisite: Strong programming skills, basic knowledge of linear algebra (L1)

---

[1] Giuseppe Di Molfetta, M Brachet, and Fabrice Debbasch. Quantum walks as massless dirac fermions in curved space-time. *Physical Review A*, 88(4):042301, 2013.

[2] Giuseppe Di Molfetta and Armando Pérez. Quantum walks as simulators of neutrino oscillations in a vacuum and matter. *New Journal of Physics*, 18(10):103038, 2016.

[3] Gregory S Engel, Tessa R Calhoun, Elizabeth L Read, Tae-Kyu Ahn, Tomáš Mančal, Yuan-Chung Cheng, Robert E Blankenship, and Graham R Fleming. Evidence for wavelike energy transfer through quantum coherence in photosynthetic systems. *Nature*, 446(7137):782–786, 2007.

[4] Salvador Elías Venegas-Andraca. Quantum walks: a comprehensive review. *Quantum Information Processing*, 11(5):1015–1106, 2012.